
Hereditarily homology-simple sets and homology critical kernels of binary images on sets of convex polytopes

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Abstract

We define a *binary image* to be a mapping $I : X \rightarrow \{0, 1\}$ in which X is a set of nonempty sets (e.g., a set of cubical voxels) in a Euclidean space and $I^{-1}[1]$ is finite: We say I is a *binary image on X* and call each element of $I^{-1}[1]$ a 1 of I . For any set S of 1s of I we use the term *S-intersection* to mean a nonempty set that is the intersection of a nonempty subset of S . Thus an *S-intersection* is either an element of S or a nonempty intersection of two or more elements of S .

Let D be any set of 1s of a binary image I . If the inclusion $U(I^{-1}[1] \setminus D) \rightarrow U I^{-1}[1]$ induces homology isomorphisms in all dimensions, then we say D is *homology-simple* in I . If every subset of D is homology-simple in I , then we say D is *hereditarily homology-simple* in I .

A local characterization of hereditarily homology-simple sets can be useful for designing parallel thinning algorithms or for checking the topological soundness of proposed parallel thinning algorithms. When I is a binary image on the grid cells of a Cartesian grid of dimension ≤ 4 , it can be deduced from results of Bertrand and Couprie that the sets D of 1s that are hereditarily homology-simple in I can be locally characterized as follows in terms of Bertrand's concept of *critical kernel*:

A set $D \subseteq I^{-1}[1]$ is hereditarily homology-simple in I if and only if every D -intersection in I 's critical kernel is a subset of a 1 of I that is not in D .

After discussing this characterization and some of its consequences, we will explain how we can generalize it to a local characterization of hereditarily homology-simple sets of 1s in any binary image I on an arbitrary set of convex polytopes of any dimension. To do this, we need only replace "I's critical kernel" in the above characterization with "I's homology critical kernel". We define the latter to be the set of all $I^{-1}[1]$ -intersections c for which the intersection of c with the union of the 1s of I that do not contain c either is empty or is disconnected or has non-trivial homology in some positive dimension.

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