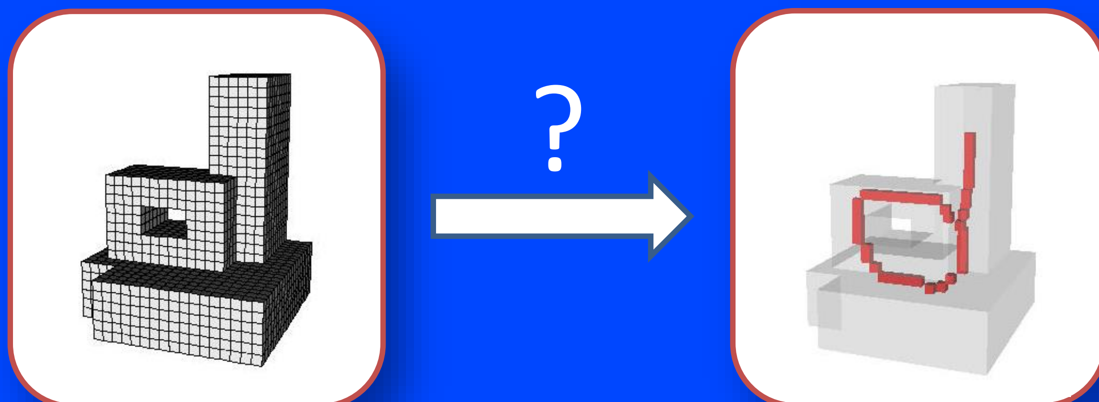
Presentation/Recall of several 3D Thinning Schemes based on Bertrand's P-simple points and critical kernels notions



Christophe LOHOU

https://christophelohou.wordpress.com/activites-de-recherche/

Institut Pascal – Clermont-Ferrand

25 mars 2019 – ESIEE Paris

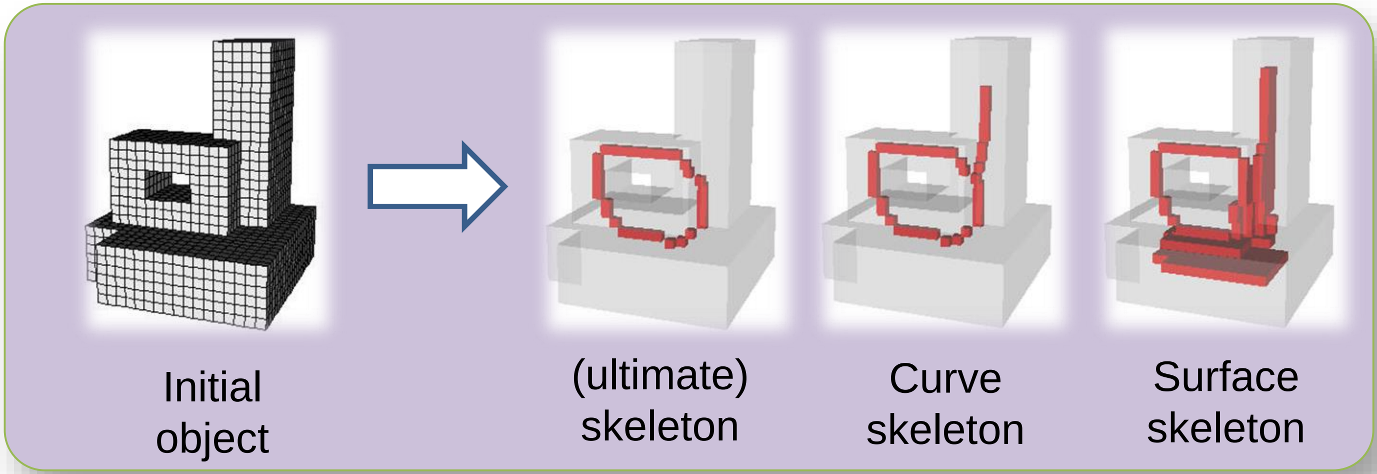
Workshop on Digital Topology and Mathematical Morphology
on the occasion of the retirement of Gilles Bertrand

Thinning process

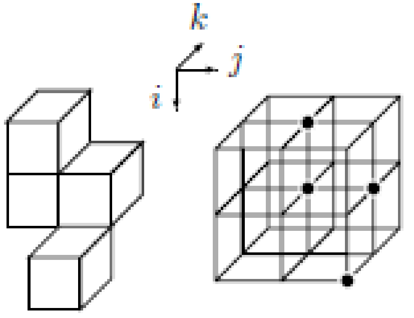
Definition

- **Thinning/skeletonization algorithm** : process which deletes points from an image while preserving its topology
  - Topological constraints: deletable points
  - Geometrical constraints: end points

Q1: Which topology framework should be used ?
Q2: How to characterize deletable/end points ?
Q3: How to design a thinning algorithm ?



DIGITAL TOPOLOGY FRAMEWORK



(a) (b)

X $X \subseteq Z^3$

C. R. Acad. Sci. Paris, t. 321, Série I, p. 1077-1084, 1995

Informatique théorique/*Computer Science*
(Théorie des signaux/*Theory of Signals*)

On *P*-simple points

Gilles Bertrand

Simple point

Definition

[Morgenthaler 1981, Kong 1989]

• Let $X \subseteq Z^3$

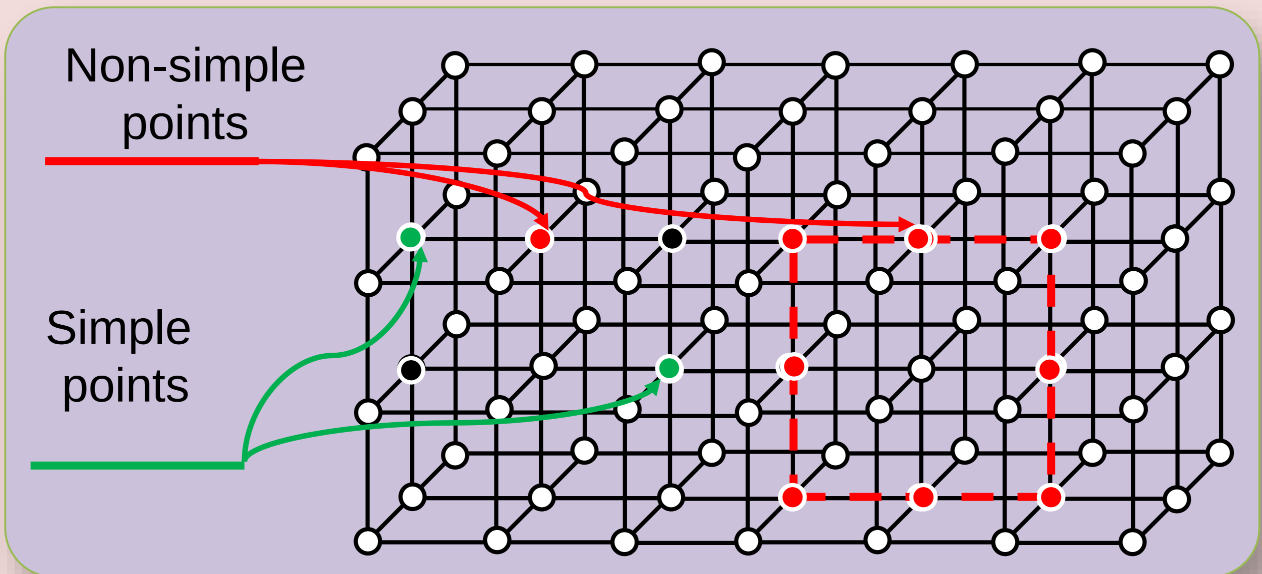
A point $x \in X$ is *simple* if

its deletion does not « change the topology of the image»,

i.e., if point x is deleted:

- the components of X and $\overline{X}$ are preserved
- the holes of X and $\overline{X}$ are preserved.

Global notion



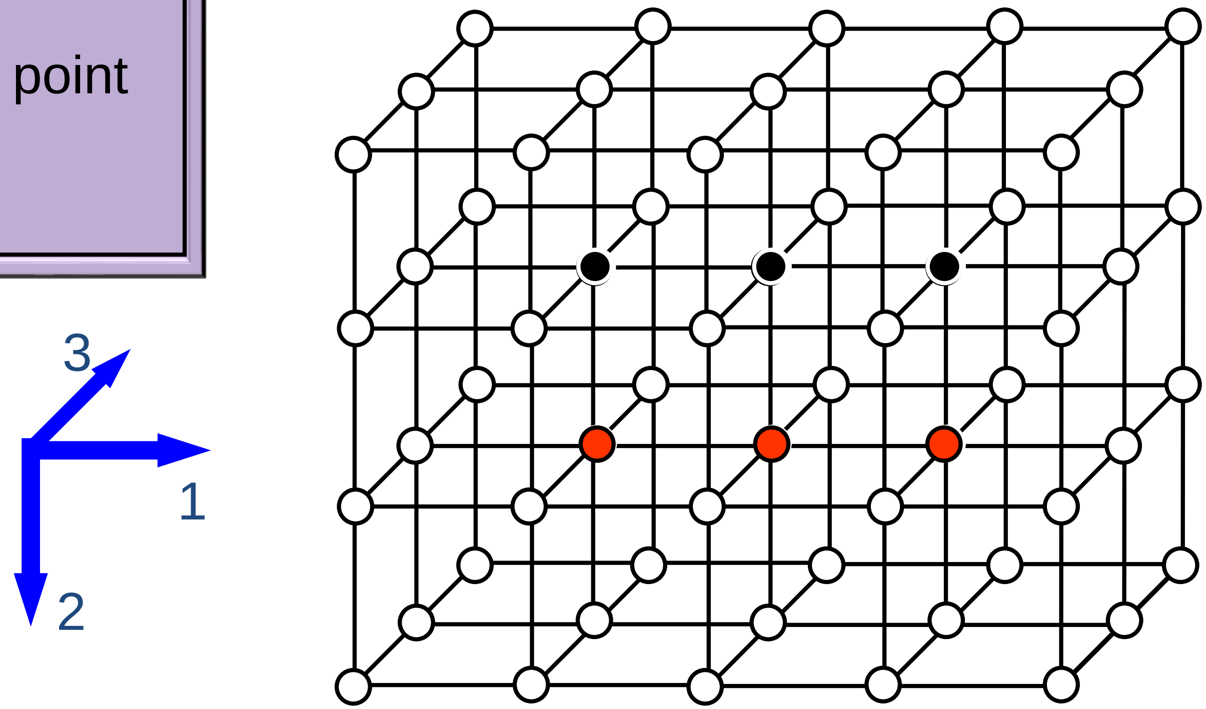
Local characterization with topological numbers
[Bertrand & Malandain, 1992, Malandain & Bertrand, 1994]

Sequential Algorithm

Repeat

delete a simple point
which is not an end point
(video scan)

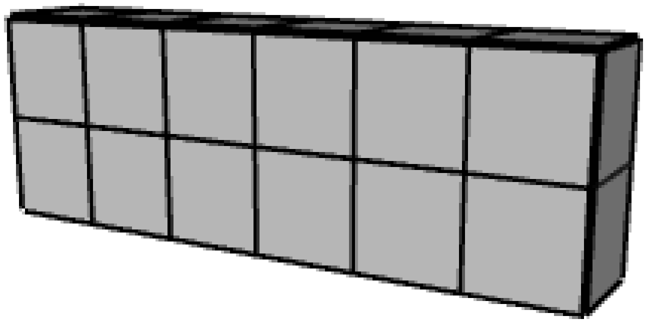
until stability



Topology preservation proof

Bad SK-centering
Bad efficiency

Problem [Parallel deletion]



Usually,
all simple points
cannot be
simultaneously
deleted!

Simple Set

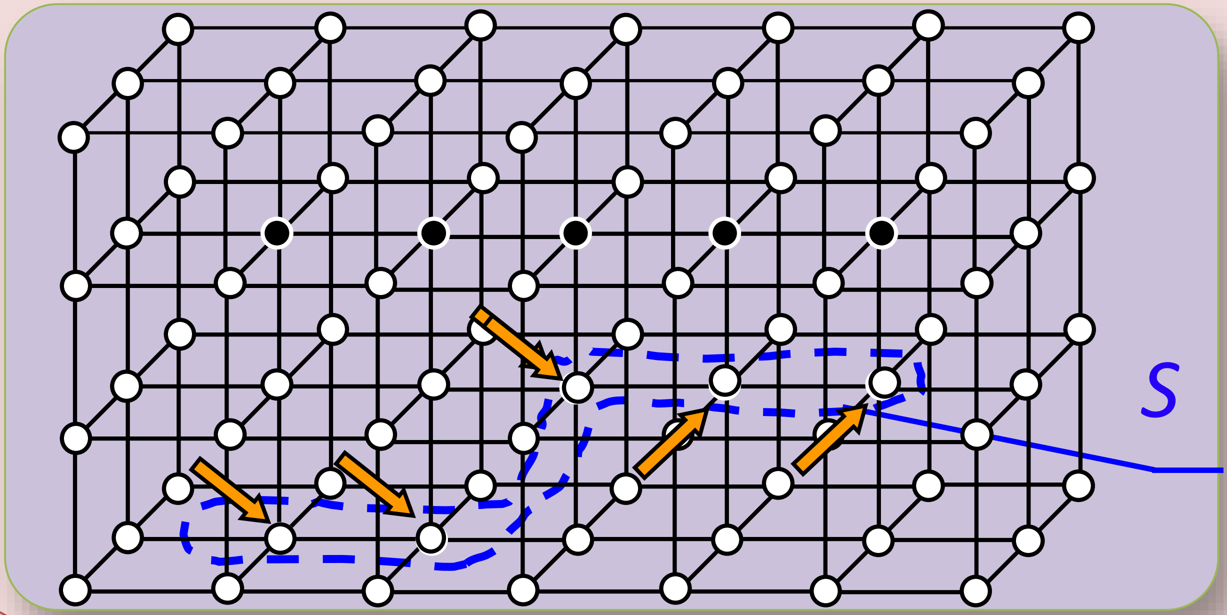
Definition [Ma, 1994]

Let $X \subseteq Z^3, S \subseteq X$

S is a *simple set of* X if points of S may be arranged according to a sequence $S = \{x_1, \ldots, x_k\}$

such that x_1 is simple for X,

and x_i is simple for the set $X \setminus \{x_1, \ldots, x_{i-1}\}$ for $i = 2, \ldots, k$



Minimal Non Simple Set

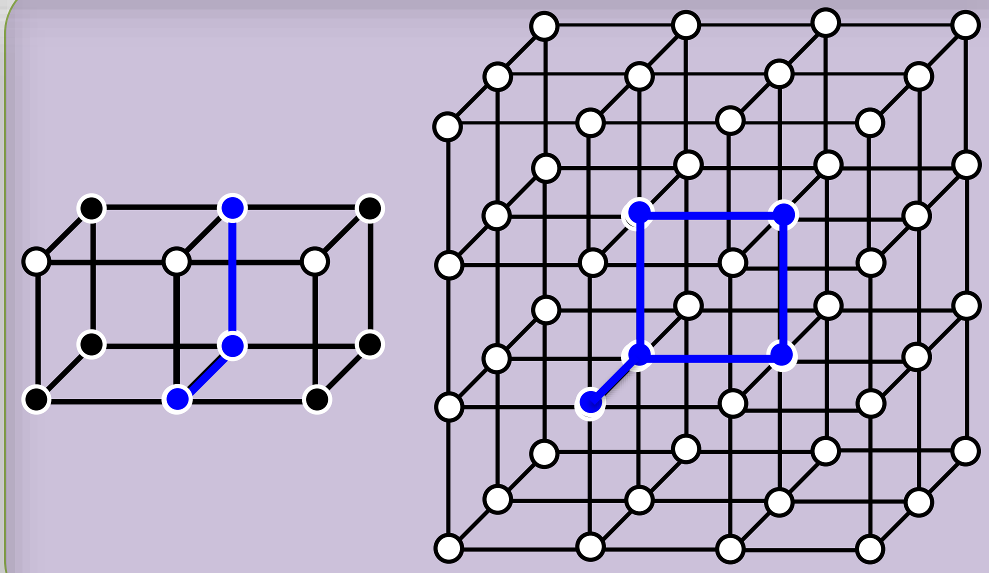
Definition [2D : Ronse 1986,1988; 3D : Hall 1992; Ma 1993,1994]

A *minimal non simple set* is a non simple set whose all subsets are simple sets.

Theorem [Ma, 1993]

Let *O* be a parallel thinning operator,
O well preserves topology if:

- every set of black points, included inside a unit square, and that is deleted by *O*, must be a simple set,
- *O* must not delete any connected component included inside a unit cube.



In practical

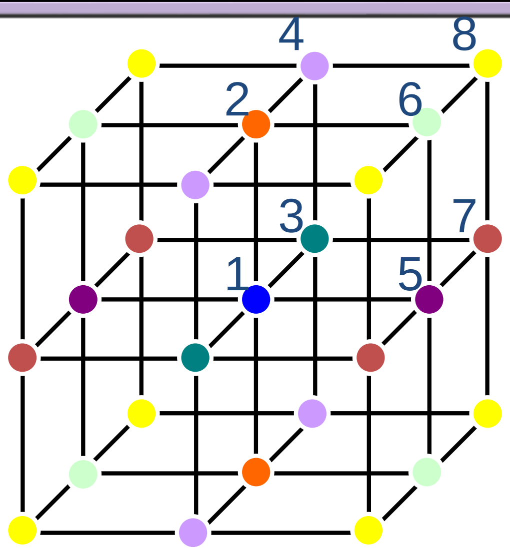
The only theorem used before
P-simple points introduction
(hard combinatory proofs)

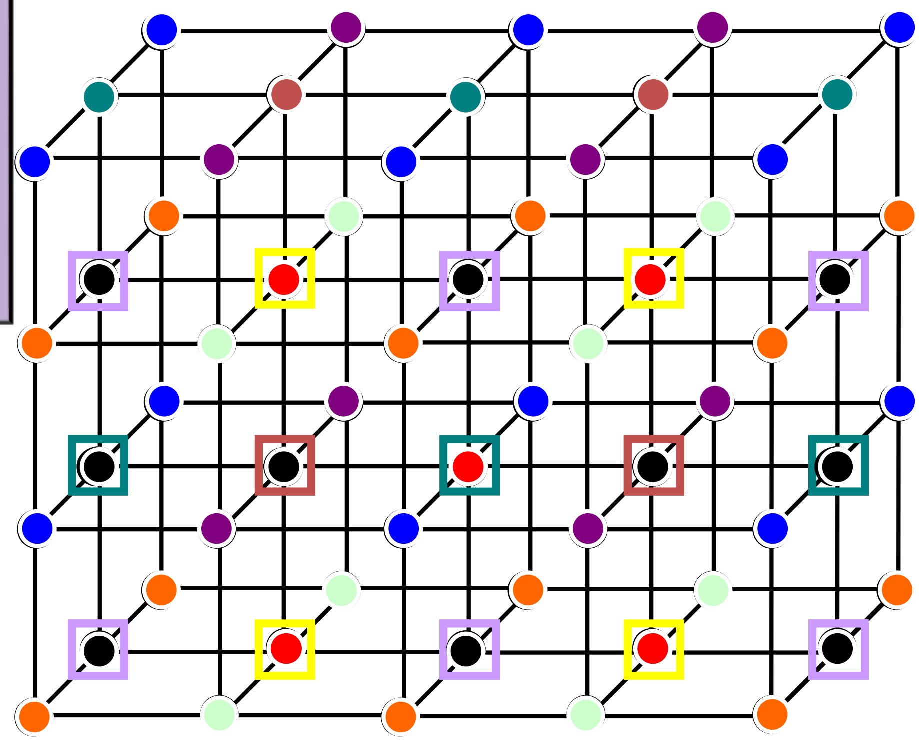
Subgrids Thinning Algorithms

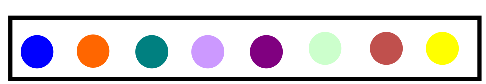
<u>Repeat</u>

For each *i* th subgrid
delete in parallel simple points
of the *i* th subgrid,
which are not end points

<u>Until</u> stability







Topology preservation proof

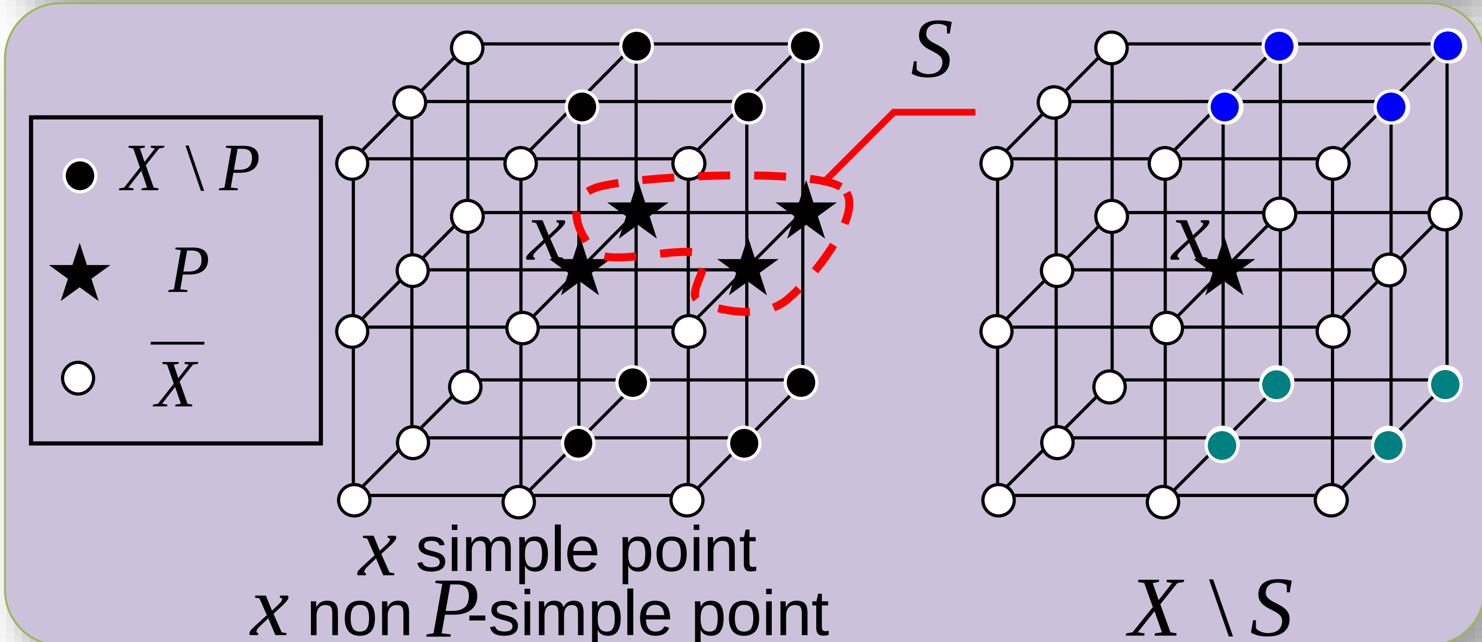
Bad SK-aspect (jagged)

P-simple Points

Definition [Bertrand, 1995]

Let $X \subseteq Z^3, P \subseteq X$ and $x \in P$

x is *P-simple* if $\forall S \subseteq P \setminus \{x\}, x$ is simple for $X \setminus S$



Property [Bertrand, 1995]

local characterization of P-simple point x

once *P* is known in *N(x)*

P-simple Points

Definition (recall)

[Bertrand, 1995]

Let $X \subseteq Z^3, P \subseteq X$ and $x \in P$

x is *P-simple* if $\forall S \subseteq P \setminus \{x\}, x$ is simple for $X \setminus S$

In practical

P={set of points which are candidate to be deleted by a parallel thinning algorithm}

Property

Any thinning algorithm that only deletes P-simple points in parallel (automatically) well preserves topology

Step back:

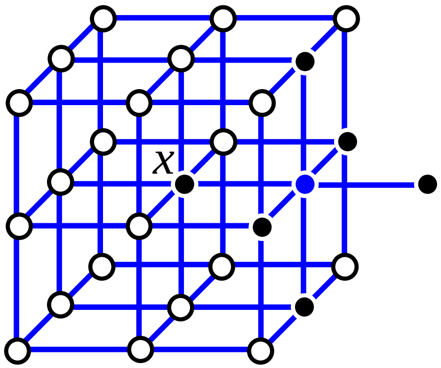
Opposite approach of thinning algorithms design: no proof is required!

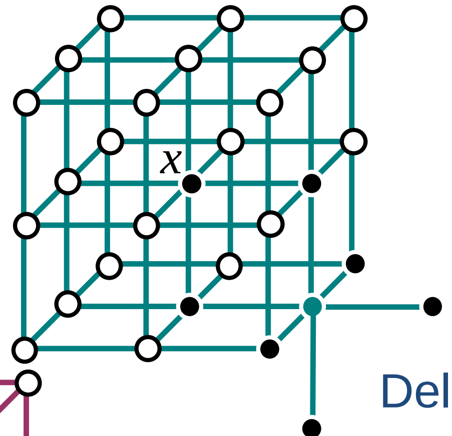
Parallel Algorithms

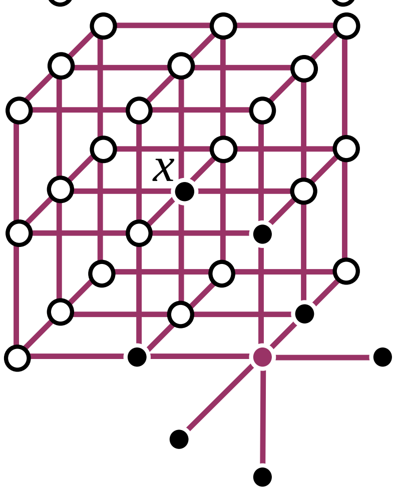
<u>Repeat</u>

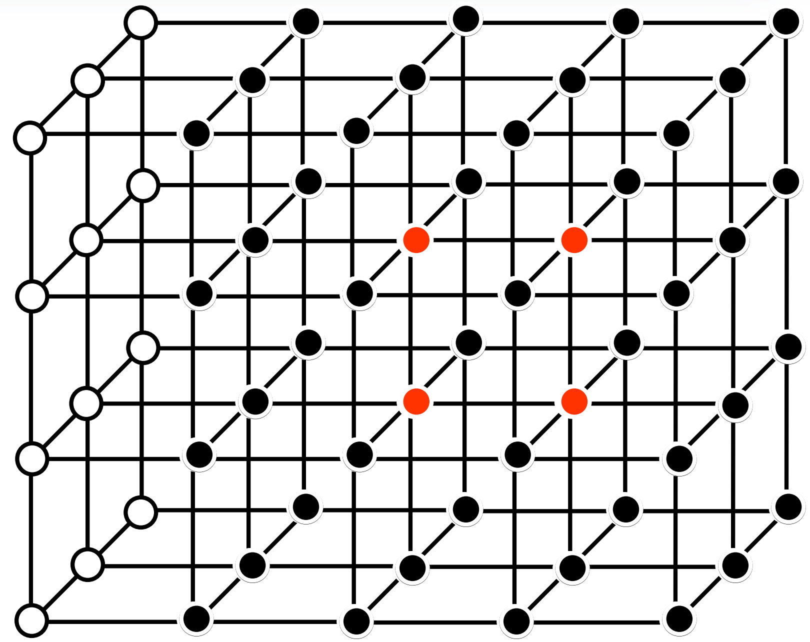
delete in parallel simple points not end points which verify deleting templates

<u>Until</u> stability









Deleting templates (up to isometries)
(symmetrical templates?)

Fully parallel:
- Ma, 1995
- Ma & Sonka, 1996

Symmetrical:
- Manzanera et al, 1999

Soundness of thinning algorithms with P-simple points

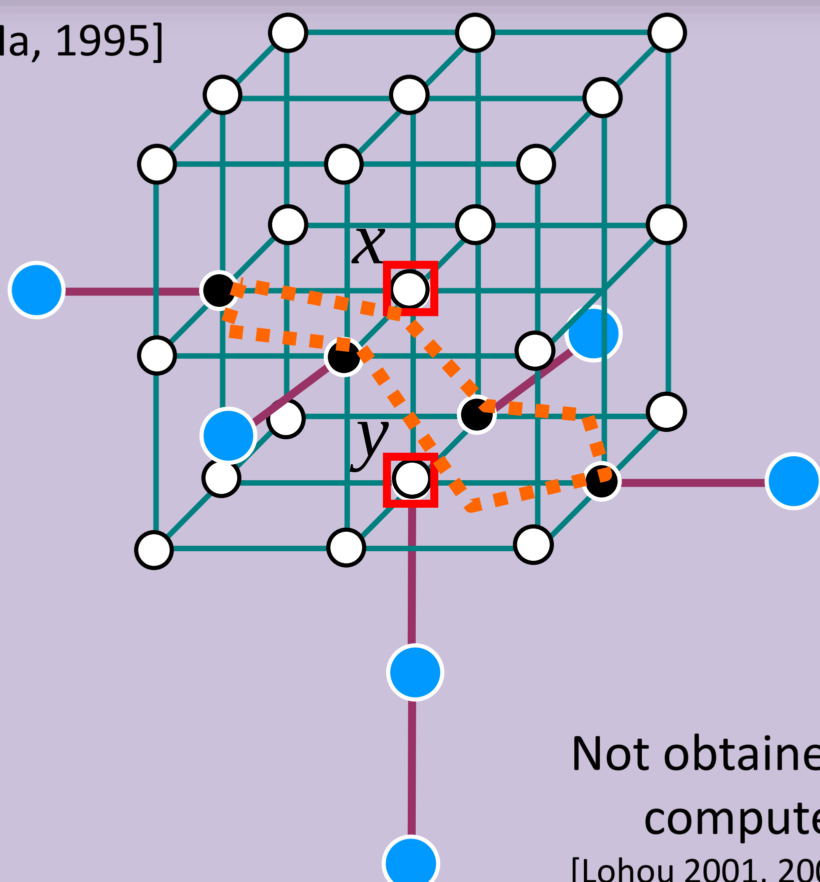
Process

[Bertrand, 1995]

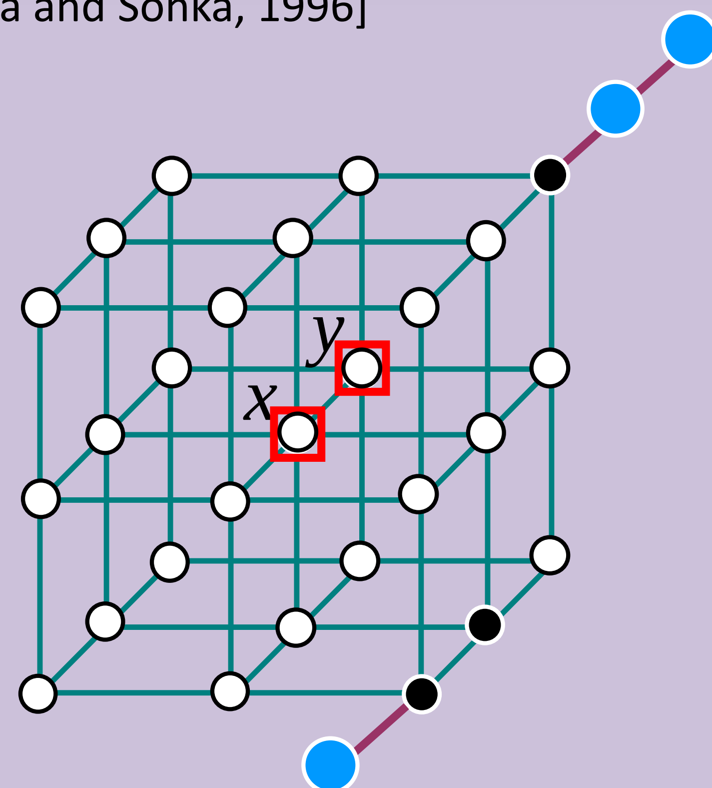
O: parallel thinning algorithm, P ={points deleted by O during one iteration}

$$\forall X \subseteq Z^3, \forall x \in P, x \text{ is } P\text{-simple} \; ?$$

[Ma, 1995]



[Ma and Sonka, 1996]



Not obtained by a computer !

[Lohou 2001, 2008, 2010]

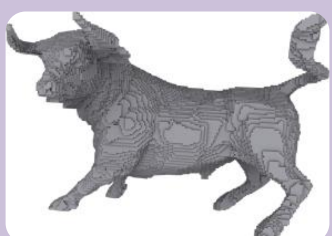
Automatic correction

Process

[Bertrand, 1995]

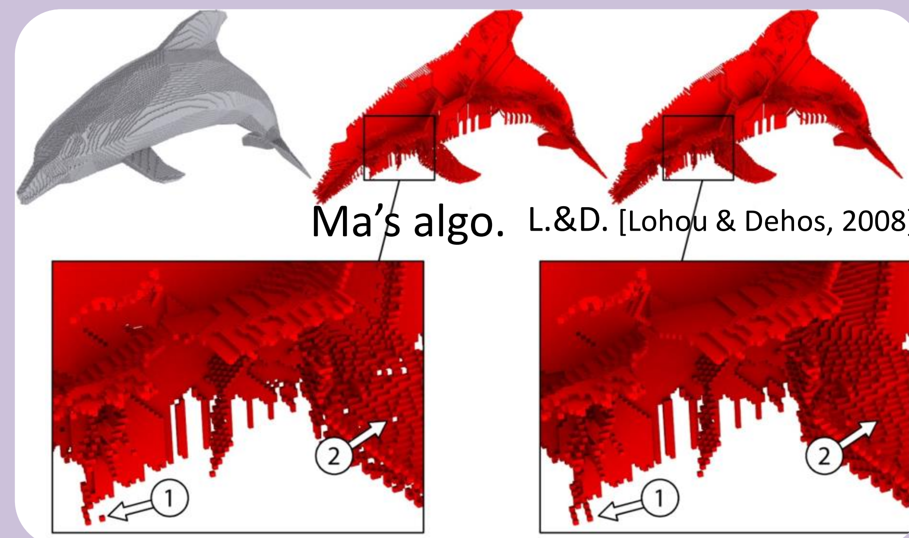
Let O be an operator; « correction » :

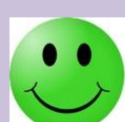
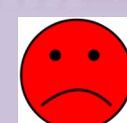
$$P = X \setminus O(X) \quad ; \quad O'(X) = X \setminus \{P\text{-simples of } X\}$$



M.&S.' algo

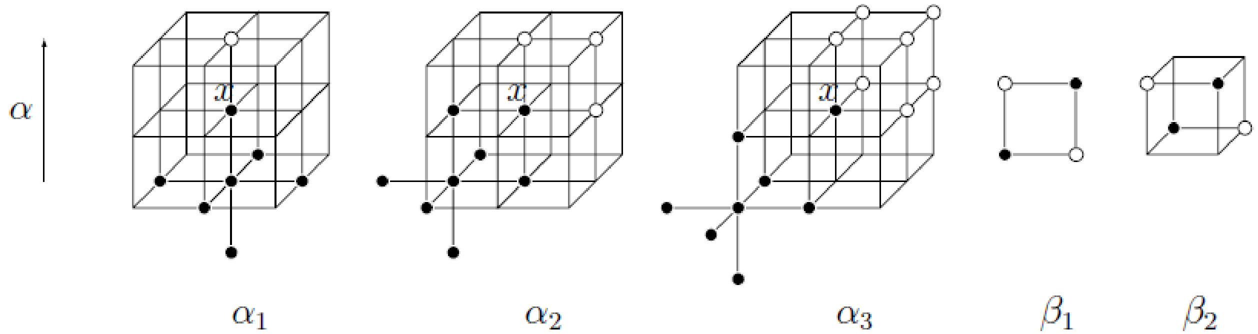
L.&D.' Correc. [Lohou & Dehos, 2010]



 Topology preservation proof  Bad efficiency

Symmetrical algorithm

- Manzanera & al, 1999



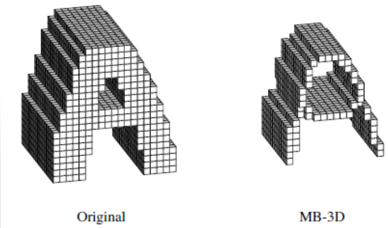
A point x is deleted if it verifies $\alpha_1, \alpha_2, \text{or } \alpha_3$, unless $\beta_1 \subseteq N_{18}(x)$ or $\beta_2 \subseteq N_{26}(x)$

Topology preservation proof

Bad SK-aspect

Question

Better control the skeleton thinness and aspect?



Symmetrical thinning algorithms with P-simple points

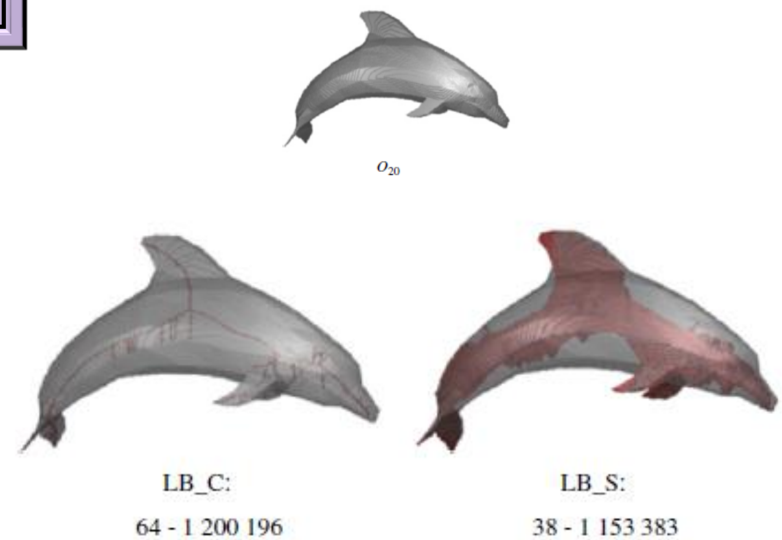
Repeat until stability

$X \leftarrow X \setminus P\text{–simple points for } X$

[Lohou & Bertrand, 2007]

$$P_S = \{x \in X \; ; \; T_6(x, \bar{X}) \neq 0\}$$

$$P_C = \{x \in X \; ; \; T_6(x, \bar{X}) = 1\} \bigcap \{x \in X \; ; \; \forall y \in N_6^*(x) \bigcap X \, , if \;\; T_6(y, \bar{X})$$



Topology preservation proof
SK-aspect

Bad efficiency
Difficult implementation

Question

Better separation deletable/end points ?

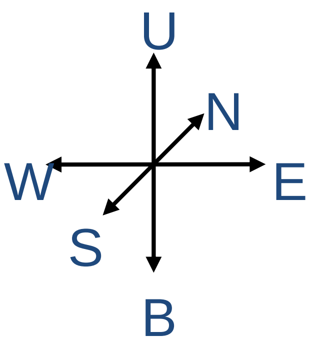
Directional Algorithms

<u>Repeat until stability</u>

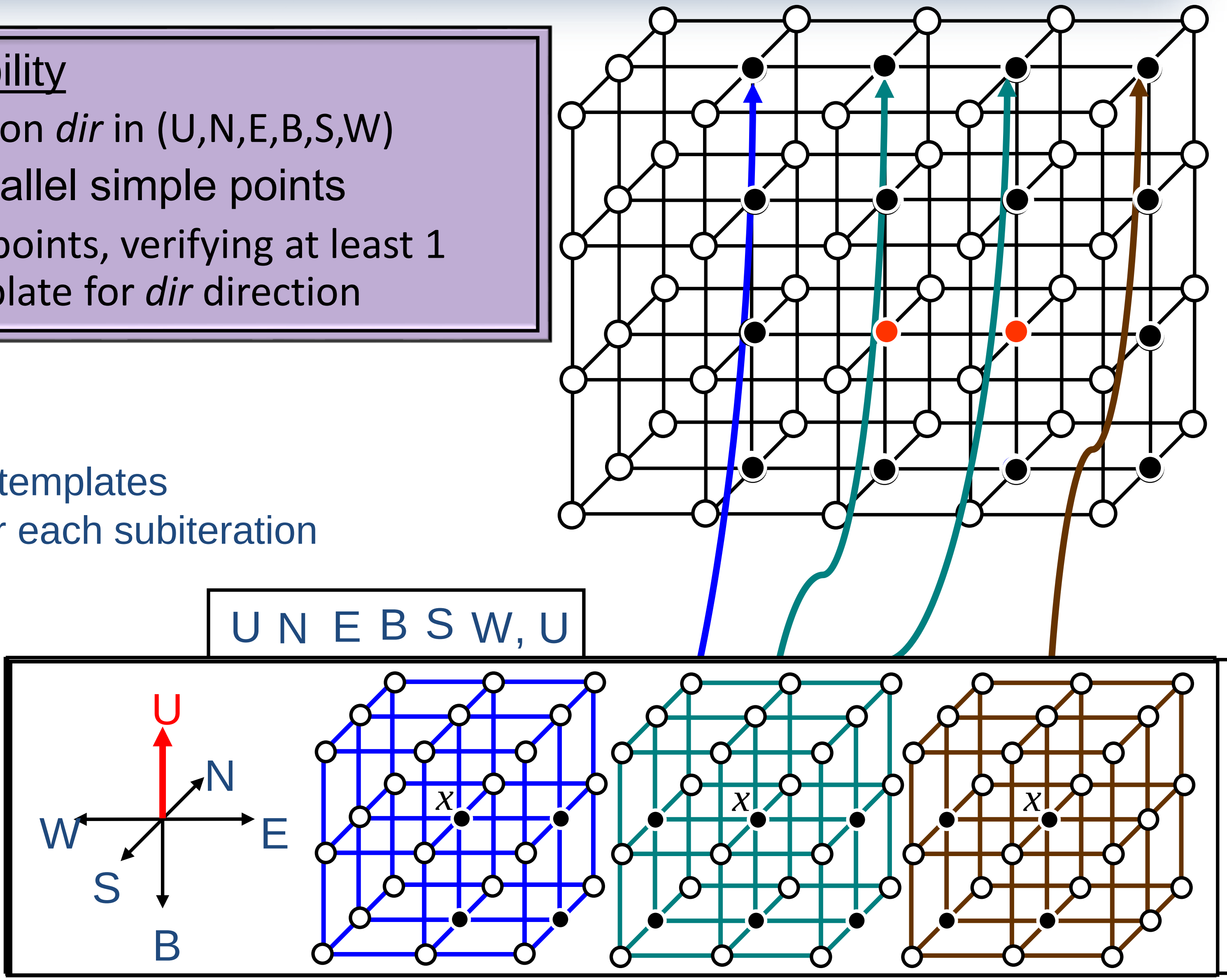
For each direction *dir* in (U,N,E,B,S,W)

delete in parallel simple points

non end points, verifying at least 1 deleting template for *dir* direction



+ templates
for each subiteration



P^x-simple-points and directional thinning, Most powerful

Definition [Lohou, 2001]

- O algorithm, P={points candidate to be deleted by O}
- Let $x \in X$, P^x : set of points which **may** verify the condition to membership to P by the only examination of $N(x) \cap X$ → P^x-simple point

Property [Lohou, 2001]

Any P^x-simple point is a P-simple point. Any operator which deletes P^x-simple points well preserves topology

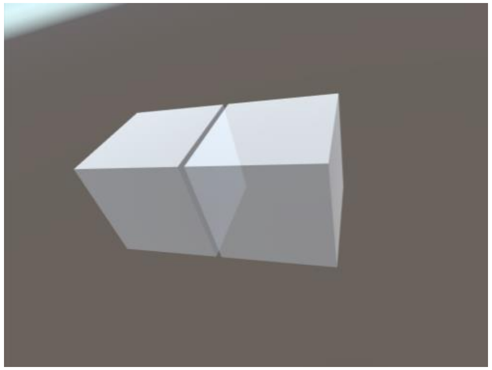
More ***powerful*** algorithm *O'* from an actual algorithm *O* (templates in *N(x)*):

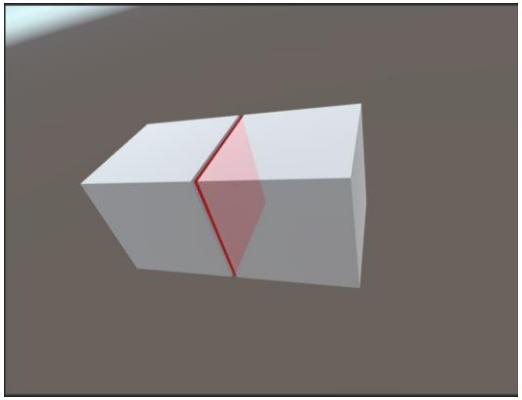
→ *O'* deletes at least all points deleted by a *O* (for a same object); *O* preserving topo.

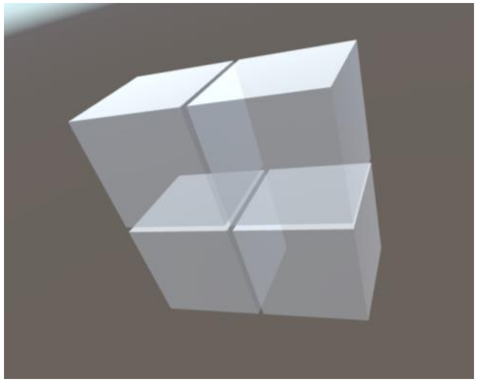
Topology preservation proof Efficiency of code

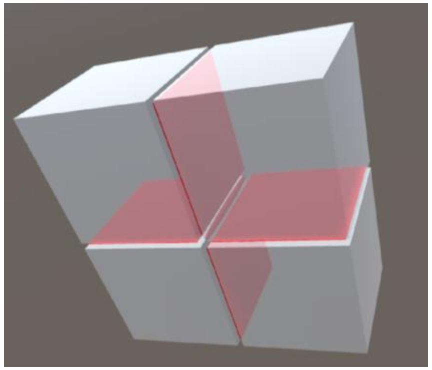
Implementation of « initial » code (iterations to design the algorithm)

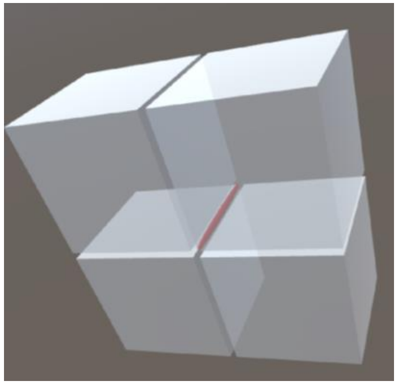
Transition to PartII

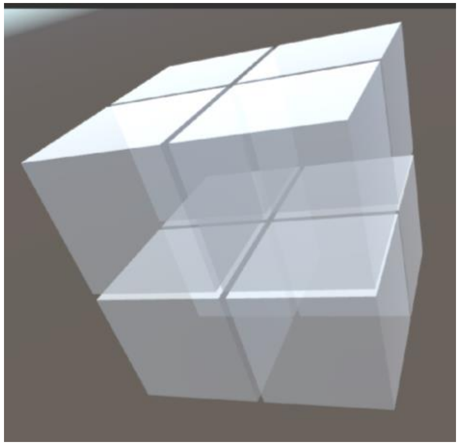


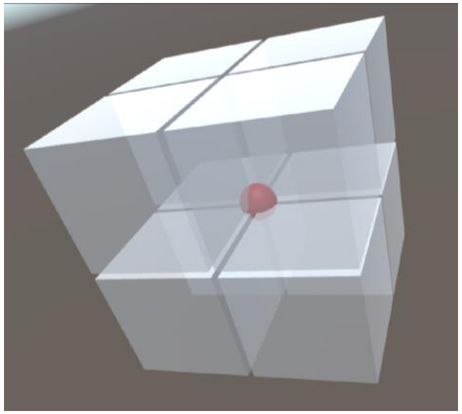












PartII – Cubical complexes

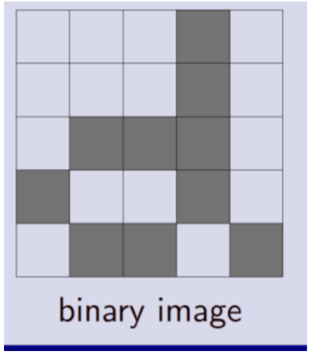
C. R. Acad. Sci. Paris, Ser. I 345 (2007) 363–367

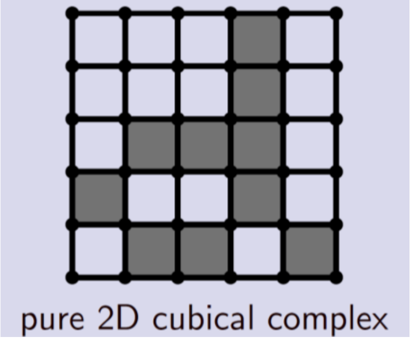
Combinatorics

On critical kernels

Gilles Bertrand

2D

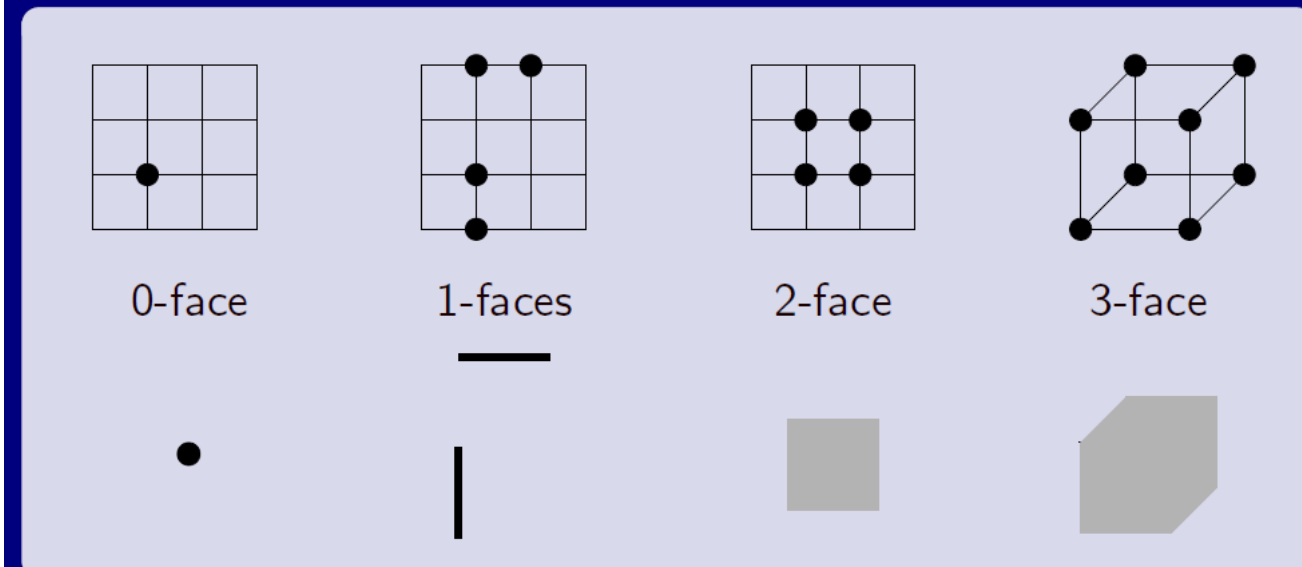




Cubical complexes

d-face

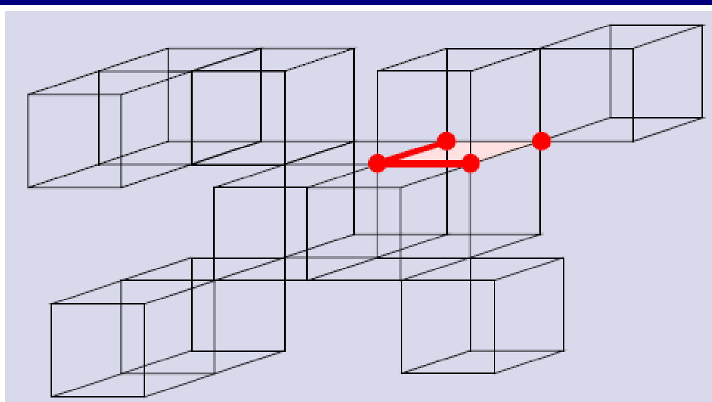
- A subset of $\mathbb{Z}^n$ composed of one point is called a 0-face.
- A subset of $\mathbb{Z}^n$ forming a unit bipoint is called a 1-face.
- A subset of $\mathbb{Z}^n$ forming a unit square is called a 2-face.
- A subset of $\mathbb{Z}^n$ forming a unit cube is called a 3-face.



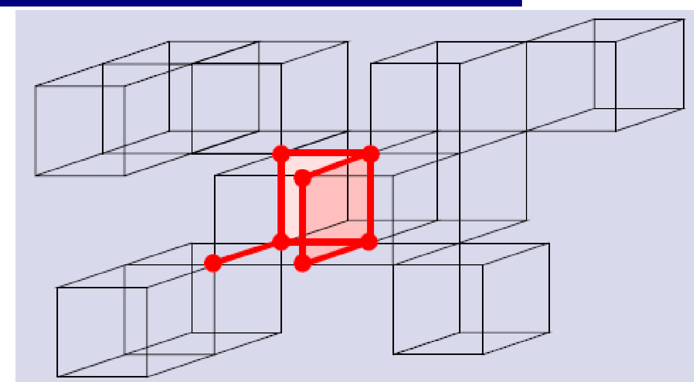
Regular/critical face

Definition

- We say that f is regular if $f \in Ess(X)$ and if $\hat{f}$ collapses onto $Core(f, X)$.
- We say that f is critical if $f \in Ess(X)$ and if f is not regular.



The 2-face is critical



The 3-face is regular

Definition

- A face f in X is a maximal critical face, or an M-critical face, if f is a critical face which is not strictly included in any other critical face.

Generic thinning scheme

Definition

- Let S be a set of n-xels and let $K \subseteq S$.
- We denote by $Cruc(S, K)$ the set composed of all n-xels which are in the crucial kernel of S or which are in K.
- Let $\langle S_0, S_1, \ldots, S_k \rangle$ be the unique sequence such that $S_0 = S$, $S_i = Cruc(S_{i-1}, K)$, $i = 1, \ldots, k$ and $S_k = Cruc(S_k, K)$.
- The set S_k is the $\mathcal{K}$-skeleton of S constrained by K.

… local characterization with templates for cliques

Results

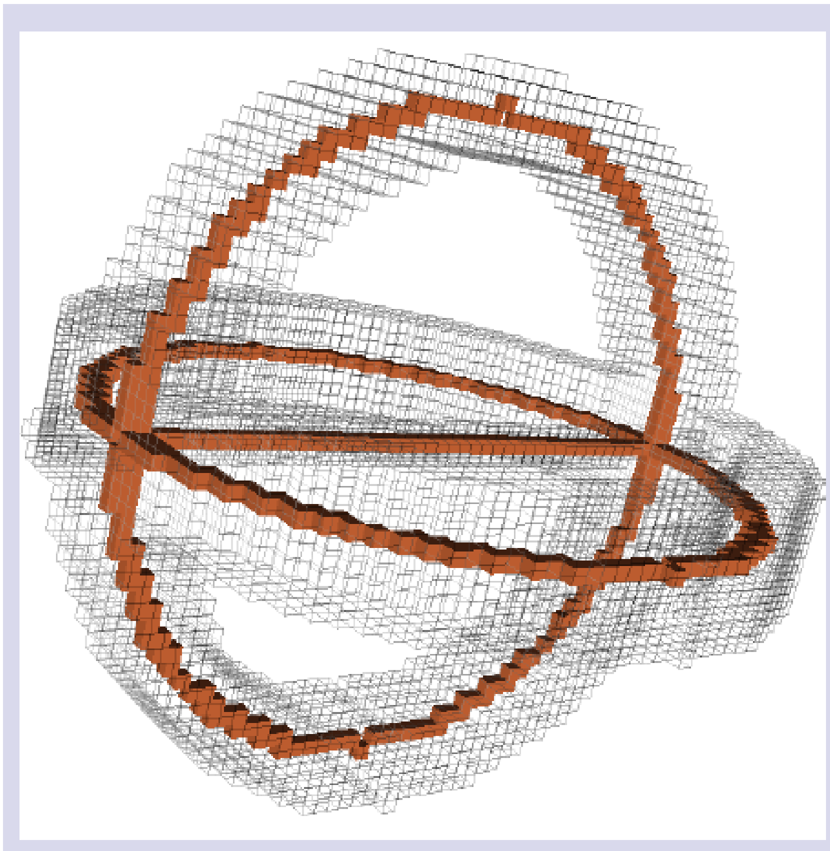
A New 3D Parallel Thinning Scheme Based on Critical Kernels

Gilles Bertrand and Michel Couprie

2014

with $K = \emptyset$

First fully // algo
for **minimal skeletons**



Skeletons are invariant by 90° rotations

… local conditions (masks)

Isthmus based parallel and symmetric 3D thinning algorithms 2015

Gilles Bertrand*, Michel Couprie

[redacted]

Algorithm 1: CrucialIsthmus(X, K, k).

Data: $X \in \mathbb{V}^3, K \in \mathbb{V}^3, k \in \{1, 2, 2^+\}$
Result: $Y = \mathcal{D}(X, K), Z = \mathcal{I}(X, K, k)$

$Y := K$;
$Z := \emptyset$;
for $d \leftarrow 3$ **downto** 0 **do**
  A := set of all voxels belonging to any d-clique that is critical for X and included in $X \setminus Y$;
  B := set of all voxels belonging to any d-clique that is k-critical for X and included in $X \setminus Y$;
  $Y := Y \cup A$;
  $Z := Z \cup B$;

Algorithm 2: IsthmusSymThinning(X, k).

Data: $X \in \mathbb{V}^3, k \in \{1, 2, 2^+\}$
Result: X

$K := \emptyset$;
repeat
  $Y = \mathcal{D}(X, K)$;
  $Z = \mathcal{I}(X, K, k)$;
  $X := Y$;
  $K := K \cup Z$;
until *stability*;

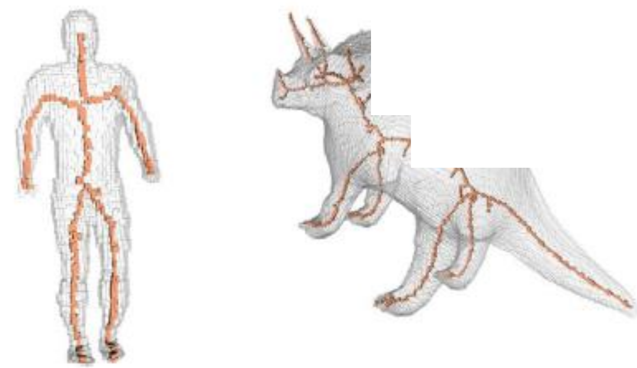


Fig. 15. Illustrations of algorithm IsthmusSymThinning, with $k = 1$.

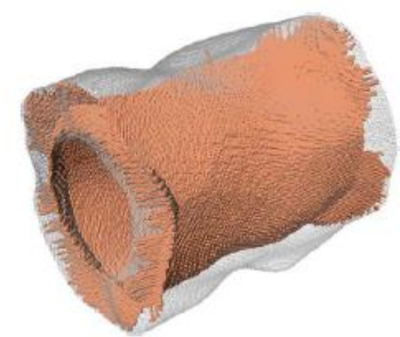


Fig. 16. Illustration of algorithm IsthmusSymThinning, with $k = 2$.

Sequential algorithms:
John Chaussard's
PhD thesis, 2010 ;
Couprie & Bertrand, 2015

Conclusion

- Digital topology …
- Critical kernels:
  - Fancy way to define/characterize
    - deletable faces … voxels
    - end points (isthmuses)
    - algorithms
  - Algorithms:
    - automatically well preserve topology (no proof)
    - easier implementation
  - Skeletons have better properties
    (separation between deletable and end points)

THANK YOU FOR YOUR ATTENTION